

Space-Like Motions of Quantum Zero Mass Neutrinos

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Recent experimental reports of super-luminal velocity neutrinos moving between Geneva and Gran Sasso in no way contradict the special relativity considerations of conventional quantum field theory. A neutrino exchanged between Geneva and Gran Sasso is both virtual and space-like. The Lorentz invariant space-like distance L and the Lorentz invariant space-like four momentum transferred ϖ between Geneva and Gran Sasso can be extracted from experimental data as will be shown in this work.

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I. INTRODUCTION

There have been recent large group experimental reports[1, 2] that neutrinos have been observed to travel at super-luminal speeds, i.e. at speeds faster than light in an approximately inertial frame of reference. The neutrinos were reported to travel at super-luminal speeds from CERN in Geneva to CNGS in Gran Sasso. Should the experimental assertions of super-luminal neutrino motion turn out to have sufficient verification, then important and deep theoretical understanding of relativistic physics will eventually accrue. Our purpose is not to probe the detailed experimental procedures leading to reports of faster than light speed neutrinos. In what follows, we merely discuss the theoretical implications of such ultra-relativistic processes.

Let us first note within the context of quantum field theory, the fact that a neutrino moves in a super-luminal manner by no means implies a breakdown of the Einstein special relativity or of Lorentz invariance. It merely means that the experimental Lorentz invariant quantities must be chosen with care. The neutrino moving between Geneva to Gran Sasso is virtual as shown in the Feynman diagram of FIG.1 in Sec.III. (i) The Lorentz invariant space-like displacement of the virtual neutrino is denoted by L . From the data $L \sim 6$ kilometer. (ii) The Lorentz invariant space-like momentum transfer carried by the virtual neutrino between Geneva and Gran Sasso is denoted by ϖ . From the data $\varpi \sim 100$ MeV/c.

In Sec.II the special relativity kinematics of super-luminal velocities are discussed. The method of extracting the Lorentz invariant space-like displacement L from experimental data is provided. The particle reactions in Geneva and in Gran Sasso are explored. In Sec.III the virtual neutrino between Geneva and Gran Sasso are visualized. The virtual neutrino exchanges a Lorentz invariant space-like virtual neutrino momentum transfer $p^2 = \varpi^2 > 0$ [We use the metric $\eta^{\mu\nu} = -1$, i.e. (+1,+1,+1,-1)]. The method of extracting the Lorentz invariant space-like ϖ from experimental data is provided. In Sec.IV the nature of the quantum wave function

of a virtual neutrino is explored.

II. SUPER-LUMINAL KINEMATICS

For a particle moving with velocity

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}, \quad (1)$$

a sub-luminal velocity defines the Lorentz invariant proper time of motion as

$$c^2 d\tau^2 = c^2 dt^2 - |d\mathbf{r}|^2 \text{ for } |\mathbf{v}| < c, \\ d\tau = dt \sqrt{1 - \left(\frac{|\mathbf{v}|}{c}\right)^2}. \quad (2)$$

For a particle moving at super-luminal velocities, the Lorentz invariant proper space of motion is

$$ds^2 = |d\mathbf{r}|^2 - c^2 dt^2 \text{ for } |\mathbf{v}| > c, \\ ds = d|\mathbf{r}| \sqrt{1 - \left(\frac{c}{|\mathbf{v}|}\right)^2}. \quad (3)$$

A. Space-Like Displacement

For a super-luminal velocity which is uniform in space and time,

$$v = \frac{|\mathbf{r}_1 - \mathbf{r}_2|}{|t_1 - t_2|} > c, \text{ with } r = |\mathbf{r}_1 - \mathbf{r}_2| \quad (4)$$

the Lorentz invariant space-like displacement is as in Eq.(3); i.e.

$$L^2 = |\mathbf{r}_2 - \mathbf{r}_1|^2 - c^2 |t_2 - t_1|^2, \\ L = r \sqrt{1 - \left(\frac{c}{v}\right)^2} = r \sqrt{\left(\frac{v+c}{v}\right) \left(\frac{v-c}{v}\right)}. \quad (5)$$

While an experimental measurement of r and v depend on that Lorentz frame of reference to be named the laboratory frame, the space-like displacement L is independent of which Lorentz frame of reference is chosen for a physics analysis.

What is a bit more subtle is the analysis of the direction of time. It is well known that Feynman[3] analyzed the notion of a particle as going forward in time and an anti-particle as going backwards in time. If the particle goes space-like with a displacement L , as in Eq.(4), then in one Lorentz frame the particle may go $1 \rightarrow 2$ and in another Lorentz frame the anti-particle may go $2 \rightarrow 1$ [4]. For example, if an experimental group reports that a super-luminal neutrino went from Geneva to Gran Sasso in some laboratory Lorentz frame of reference, then in another Lorentz frame of reference a super-luminal anti-neutrino went from Gran Sasso to Geneva. There is simply no Lorentz invariant way to time order space-like separated events.

All of these considerations do not require that Einstein be abandoned. One can in a relativistic invariant way simply quote the Lorentz invariant space-like displacement L . The reported super-luminal European neutrino or anti-neutrino transport asserts a space-like displacement of $L \sim 6$ kilometer with values depending on laboratory distance scales $r \sim 730$ kilometer and laboratory neutrino energy.

It is crucially important to discuss these matters in terms of physical quantities which are Lorentz invariants. Sub-luminal particles are described in terms of Lorentz invariant quantities such as mass m determined by $p^2 = -m^2c^2$. One should not forget that energy E is merely one component of a four momentum $p_\mu = (\mathbf{p}, -E/c)$. When particles or anti-particles are super-luminal, the four momentum definition perhaps becomes somewhat subtle but the requirement of a Lorentz invariant description remains sacrosanct.

B. Particle Reactions

Consider the decay of an anti-pion into an anti-muon plus a muon neutrino,

$$\pi^+ \rightarrow \mu^+ + \nu_\mu. \quad (6)$$

The neutrino is detected by a nuclear weak interaction via

$$\nu_\mu + N_i \rightarrow N_f + \mu^- \quad (7)$$

Altogether, the reaction amounts to the creation of a muon pair changing the internal state of the nucleus N ; i.e.

$$\pi^+ + N_i \rightarrow N_f + \mu^+ + \mu^-. \quad (8)$$

There is no neutrino in Eq.(8) because the neutrino is merely virtual. In a profound Appendix B in a paper[5]

wherein Feynman gave a derivation from quantum field theory of the diagram rules, he asserts that what looks real on a short time scale appears virtual on a long time scale. Let us apply this discussion to the muon pair production Eq.(8) with the μ^+ in Geneva and the μ^- in Gran Sasso. It appears clear that a neutrino or anti-neutrino had to make the cross country trip whether or not one considers such neutrino or anti-neutrino motions as real or virtual.

III. REAL AND VIRTUAL PROCESSES

Let us first consider the neutrino or anti-neutrino four momentum p . Regarded as a real pion decay process in Eq.(6), the four momentum

$$p = p(\pi^+) - p(\mu^+) \quad (9)$$

Regarded as a real neutrino absorption in Eq.(7), the four momentum

$$p = p(N_f) + p(\mu^-) - p(N_i). \quad (10)$$

Equating the above two expressions for the momentum p yields the conservation of four momentum pair production Eq.(8) for which

$$p(\pi^+) + p(N_i) = p(N_f) + p(\mu^+) + p(\mu^-). \quad (11)$$

While it is nice that four momentum in Geneva + Gran Sasso is all conserved, that fact alone does not answer questions concerning a neutrino that no longer appears in the momentum balance.

A. Classical Neutrino Four Momenta

For a classical *real zero mass neutrino* the four momentum obeys

$$p^2 = |\mathbf{p}|^2 - (E/c)^2 = 0 \quad \Rightarrow \quad E = c|\mathbf{p}|. \quad (12)$$

The velocity based on the classical Hamilton equation

$$\mathbf{v} = \frac{\partial E}{\partial \mathbf{p}} = \frac{c\mathbf{p}}{|\mathbf{p}|} = c\mathbf{n} \quad \Rightarrow \quad v = |\mathbf{v}| = c. \quad (13)$$

Thus, a classical zero mass particle moves at light speed. This no longer holds true in quantum mechanics wherein a zero mass particle can move off the light cone.

B. Quantum Neutrino Propagation

The Schwinger action[6] corresponding to a massless neutrino source spinor η reads

$$W = \hbar \int \int \bar{\eta}(x) S(x-y) \eta(y) d^4x d^4y, \quad (14)$$

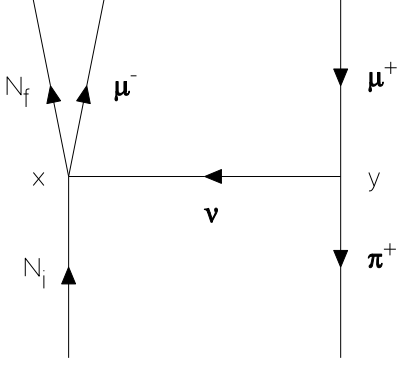


FIG. 1: Shown is the lowest order neutrino exchange diagram between an event in Geneva at y and an event in Gran Sasso at x according to Eqs.(6), (7) and (8). The virtual neutrino exchange may be space-like; i.e. $(x - y)^2 > 0$.

wherein the zero mass spin one-half particle propagator $S(x - y)$ obeys

$$-i\gamma^\mu \partial_\mu S(x - y) = \delta(x - y). \quad (15)$$

The one neutrino exchange amplitude for the particle interaction Eq.(8) is then

$$\mathcal{A}(\pi^+ + N_i \rightarrow N_f + \mu^+ + \mu^-) = i \int d^4x \int d^4y \langle N_f \mu^- | \bar{\eta}(x) | N_i \rangle S(x - y) \langle \mu^+ | \eta(y) | \pi^+ \rangle, \quad (16)$$

corresponding to the Feynman diagram in FIG.1. Solving Eq.(15) in the form

$$S(x - y) = i\gamma^\nu \partial_\nu D(x - y), \quad (17)$$

yields the propagator $D(x - y)$ obeying

$$-\partial^\mu \partial_\mu D(x - y) = \delta(x - y). \quad (18)$$

Upon Fourier transformation and with the Feynman virtual particle boundary conditions

$$\begin{aligned} D(x - y) &= \int \tilde{D}(Q) e^{iQ \cdot (x - y)} \frac{d^4Q}{(2\pi)^4}, \\ \tilde{D}(Q) &= \frac{1}{Q^2 - i0^+}, \\ D(x - y) &= \frac{i}{4\pi^2} \left(\frac{1}{(x - y)^2 + i0^+} \right). \end{aligned} \quad (19)$$

With the notation for principal part left implicit, one may write

$$\begin{aligned} \tilde{D}(Q) &= \frac{1}{Q^2} + i\pi\delta(Q^2), \\ D(x - y) &= \frac{1}{4\pi^2} \left[\pi\delta((x - y)^2) + i \left(\frac{1}{(x - y)^2} \right) \right]. \end{aligned} \quad (20)$$

The neutrino four momentum in the diagram of FIG.1 is $p = \hbar Q$.

Note that the part of the propagator wherein the neutrino is off the energy shell, i.e. $p^2 \neq 0$ and $E \neq \pm c|\mathbf{p}|$, gives rise to motions on the light cone i.e. $(x - y)^2 = 0$ and $ct = \pm c|\mathbf{r}|$. In detail, with $x - y = (\mathbf{r}, ct)$,

$$\begin{aligned} \int \left[\frac{e^{iQ \cdot (x - y)}}{Q^2} \right] \frac{d^4Q}{(2\pi)^4} &= \frac{\delta((x - y)^2)}{4\pi} = \\ \frac{\delta(r^2 - c^2t^2)}{4\pi} &= \frac{1}{8\pi r} [\delta(r - ct) + \delta(r + ct)]. \end{aligned} \quad (21)$$

Similarly, the part of the propagator wherein the neutrino is on the energy shell, i.e. $p^2 = 0$ and $E = \pm c|\mathbf{p}|$, gives rise to motions off the light cone i.e. $(x - y)^2 \neq 0$ and $ct \neq \pm c|\mathbf{r}|$. In detail

$$\begin{aligned} \int \delta(Q^2) e^{iQ \cdot (x - y)} \frac{d^4Q}{(2\pi)^4} &= \frac{1}{4\pi^3(x - y)^2} = \\ \frac{1}{4\pi^3(r^2 - c^2t^2)} &= \frac{1}{4\pi^3} \left(\frac{1}{L^2} \right). \end{aligned} \quad (22)$$

The uncertainty principle $\Delta E \Delta t > \hbar/2$ forbids the neutrino to be *both* on the energy shell $p^2 = 0$ and on the light cone $(x - y)^2 = 0$.

From Eqs.(21) and (22), it follows that neutrinos can move on the forward and backward light cones. It can move on time-like and space-like intervals. The quantum theory goes considerably beyond the classical causal retarded in time viewpoint employing the forward light cone

$$D_{\text{ret}}(\mathbf{r}, t) = \frac{1}{4\pi r} \delta(r - ct). \quad (23)$$

The quantum propagator can be computed from the retarded propagator by an analytic continuation in the imaginary time domain

$$\begin{aligned} \Delta(\mathbf{r}, t^2) &= \frac{i}{\pi} \int_0^\infty \frac{t' D_{\text{ret}}(\mathbf{r}, t') dt'}{t'^2 - t^2} \quad \text{if } \Im m(t^2) < 0, \\ D(x - y) &= \Delta(\mathbf{r}, t^2 - i0^+) \quad \text{for } (x - y) = (\mathbf{r}, ct). \end{aligned} \quad (24)$$

The above method for computing acausal quantum propagation from classical causal propagation has wide validity[7].

C. Virtual Neutrino Four Momentum

For the neutrino exchange diagram in FIG.1 when employed in four momentum space, the virtual neutrino can have a space-like four momentum,

$$p^2 = |\mathbf{p}|^2 - (E/c)^2 = \varpi^2 > 0, \quad (25)$$

so that the energy of a virtual neutrino + or virtual anti-neutrino - has the form

$$E = \pm c\sqrt{|\mathbf{p}|^2 - \varpi^2}. \quad (26)$$

The velocity of the virtual neutrino

$$\mathbf{v} = \frac{\partial E}{\partial \mathbf{p}} \Rightarrow \mathbf{p} = \frac{E\mathbf{v}}{c^2} \Rightarrow |\mathbf{v}| > c. \quad (27)$$

Finally,

$$\begin{aligned} L \text{ and } \varpi &\text{ are Lorentz invariant,} \\ |\mathbf{r}| \text{ and } |\mathbf{p}| &\text{ are Lorentz frame dependent,} \\ \frac{L}{\varpi} &= \frac{|\mathbf{r}|}{|\mathbf{p}|}. \end{aligned} \quad (28)$$

Typically[1, 2] the reported values of the Lorentz invariants should read $L \sim 6$ kilometer and $\varpi \sim 100$ MeV/c in accordance with Eqs.(5) and (28).

IV. THE NEUTRINO WAVE FUNCTION

In terms of the Feynman scattering amplitude we may discuss the neutrino wave function $\psi(x)$. The neutrino wave function at space-time point x due to a source event at space-time point y is given by the amplitude in FIG.1 or equivalently in Eq.(16) as

$$\psi(x) = \int S(x-y) \langle \mu^+ | \eta(y) | \pi^+ \rangle d^4 y. \quad (29)$$

Employing Eq.(17), the effective source

$$s(y) = i\gamma^\mu \partial_\mu \langle \mu^+ | \eta(y) | \pi^+ \rangle, \quad (30)$$

and integrating Eq.(29) by parts yields

$$\psi(x) = \int D(x-y) s(y) d^4 y. \quad (31)$$

Note that the neutrino wave function obeys the massless Dirac equation only away from the source; i.e. invoking Eq.(30),

$$\begin{aligned} -i\gamma^\mu \partial_\mu \psi(x) &= \langle \mu^+ | \eta(x) | \pi^+ \rangle, \\ (i\gamma^\nu \partial_\nu)(-i\gamma^\mu \partial_\mu) \psi(x) &= -\partial^\mu \partial_\mu \psi(x) = s(x), \end{aligned} \quad (32)$$

again leading to Eq.(31).

V. CONCLUSION

The reported super-luminal velocity neutrinos moving between Geneva and Gran Sasso do not imply a breakdown of the special relativity considerations of conventional quantum field theory if the neutrino motion between Geneva and Gran Sasso is regarded as being virtual. The two Lorentz invariant quantities describing the virtual neutrino are as follows: (i) The space-like interval L traveled by the virtual neutrino is given by

$$L = |\mathbf{r}| \sqrt{1 - \left(\frac{c}{|\mathbf{v}|}\right)^2}. \quad (33)$$

(ii) Even for a massless neutrino, the four momentum of the neutrino can be space-like. Thereby the Lorentz invariant four momentum square obeys

$$p^2 = \varpi^2 > 0. \quad (34)$$

For the experimental reports at hand, the virtual neutrino travels $L \sim 6$ kilometer with a momentum transfer of $\varpi \sim 100$ MeV/c.

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